1 Introduction

1.2 *How did the political turmoil in 1930s and 1940s Europe influence the relations between the great physicists Max Planck, Albert Einstein, Niels Bohr, Werner Heisenberg, and John von Neumann, as well as their lives?*[1]

Discussion points:

Important events in Europe in the 1930s and 1940s:

In 1933, Hitler and a Nazi government came to power in Germany. In 1939, Germany invaded of Poland and the Second World War began. The war ended in 1945 with the capitulation of Germany (May) and then Japan (August).

Niels Bohr continued to work in Copenhagen after the Nazi occupied Denmark in 1940, but in 1943 he escaped to Sweden. He spent the last two years of the war in England and then in the Unites States where he was associated with the Manhattan Project.

Albert Einstein renounced his German citizenship in 1933 for political reasons and emigrated to the United States. In 1934 he published his, now-famous, paper with Podolski and Rosen[2] in which the concept of entanglement was used to question the completeness of the quantum theory.

In August 1939, he wrote a letter to President Roosevelt advising him about the discovery of nuclear fission, the possibility of building extremely powerful bombs based on this phenomenon, and urging the President to support experimental work in this direction.

Werner Heisenberg remained in Germany after Hitler came to power watching the decline of the scientific activity around him; many of his colleagues were fired by the Nazis and emigrated to the United States or other countries. Heisenberg was attacked in SS papers for his apparent sympathy for his Jewish colleagues. In 1939 he was called up for military service. At the time he was the leading physicist in Germany and the Nazis expected him to help develop a nuclear bomb. In 1941, Heisenberg and his colleagues built a nuclear reactor. After the war it became apparent that Heisenberg did not know how to make a nuclear bomb. Transcribed conversations recorded secretly at Farm Hall, near Cambridge, England where Heisenberg and other German scientists were being detained in 1945 by Allied military and intelligent services, show that he failed to grasp the limiting condition of an explosive chain reaction until after Hiroshima.[3] He maintained long after the war ended that while visiting Bohr in occupied Denmark he tried to convey the message that, deliberately, he was not going to pursue research and development of a nuclear device in Germany. The controversy about what role he played in the Nazis attempt to build a nuclear bomb is still not resolved.

John von Neumann came to the United States in the early 1930s not as a war refugee, but with an appointment at the Institute for Advanced Studies at Princeton. In 1933 he resigned the academic position he had in Germany. He was deeply involved in the Manhattan Project and in the development of the first electronic computer.

Max Planck felt that it was his duty to remain in his country during the Nazi regime, but was openly opposed to the Nazis policies. One of his sons was executed by the Nazis for his part in a failed attempt to assassinate Hitler in 1944.

[1] See A. D. Aczel.*Entanglement: The Greatest Mystery in Physics.* Four Walls Eight Windows Publishing House, New York, N.Y., 2001.

[2] A. Einstein, B. Podolsky, and N. Rosen. *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* Physical Review, 47:777, 1935.

[3] J. Bernstein and D. Cassidy. *Hitler's Uranium Club: The Secret Recordings at Farm Hall,* Copernicus Books; 2nd Rev. edition, 2001.

1.3 *Present and critically analyze the main ideas and concepts related to "positivism", "reductionism", and "holism", as approaches to explaining physical phenomena.* (See D. Deutsch. *The Fabric of Reality.* Penguin Books, New York, N.Y.,1997, pages 4-28).

Discussion Points:

Positivism (or logical positivism) is an extreme form of instrumentalism; it hold that all statements other than those describing or predicting observations are not only superfluous but meaningless. It was the prevailing theory of scientific knowledge during the first half of the twentieth century.
The scientific method involves postulating a new theory to explain some class of phenomena and then performing a crucial experimental test for which the old theory and the new theory predict different observable outcomes. The outcome of the crucial test helps to decide between the two theories. The experimental testing is not the only process involved in the growth of scientific knowledge. The overwhelming majority of theories are rejected, most often without testing them, because they contain bad explanations. An instrumentalist thinks that science is about predicting things

Reductionism: a view of the nature of science; science allegedly explains things reductively by analyzing them into components. For example, the mechanical properties of a wall are explained by regarding the wall as a vast aggregation of interacting molecules. The properties of the molecules are explained in terms of their constituent atoms which interact with each other, and so on down to the smallest particle and the most basic force. The foundation stone would be a reductive "theory of everything", a universal theory of particles, forces, space and time, together with some theory of what the initial state of the universe was. Reductionism is a mistaken view of the nature of science held by many scientists (unfortunately). At present we have only approximations to a reductive "theory of everything". The accurate laws of motion known for individual subatomic particles are supplemented with various approximations schemes in order to be able to predict some aspects of gross behavior of quite large objects. The reason why high-level subjects can be studied at all is that under special circumstances the complex behavior of vast numbers of particles resolves itself into a measure of simplicity and comprehensibility. High-level simplicity emerges from low-level complexity. A reductionist thinks that science is about analyzing things into components

Holism is the opposite of reductionism and supports the idea that the only legitimate explanations are in terms of high-level systems. Considered as an even greater error than reductionism, it does not recognize the importance of reductive explanations.

1.4 *What is the justification of the statement in* D. Deutsch. *The Fabric of Reality.* Penguin Books, New York, N.Y.,1997, page 197, *"Complexity theory has not yet been sufficiently well integrated with physics to give many quantitative answers"?*

Discussion Points:

The computational complexity theory is the study of resources that are required to perform given computational tasks.

The laws of physics are, thus far, capable of being successively approximated by theories that give better explanations and predictions. The task of discovering each theory, given the previous one, has been computationally tractable, given the previously known laws and the previously available technology. The complexity theory has made progress in defining a useful, rough-and-ready distinction between *tractable* and *intractable* computational tasks related to simulating whole physical systems based on the most general laws of physics.

1.5 *Critically analyze Fredkin's idea that the universe is a gigantic digital computer processing information* [4] *Note that the physicist Philip Morrison remarked "the only reason Fredkin thought that the universe was a computer was because he was a computer scientist, in the same way that if he had been a cheese maker, he would have claimed that it was made out of cheese"*[5]

Discussion Points:

Fredkin proposes *Digital Mechanics* as a model of physics based on the *Finite Nature* assumption. According to him "*Finite Nature* is the name of the hypothesis that space and time, momentum and energy, position, velocity and acceleration, and everything else is, in final analysis, discrete and finite. Finite Nature rules out truly random numbers that do not depend on anything for their values." He takes the position that "the basic substrate of physics operates in a manner similar to the workings of certain specialized computers called Cellular Automata. This means that a volume of space has a certain amount of computational ability." Space could be divided into cells (at the smallest scale of length) and at each instant of time each cell would be in one of a few distinguishable states. They would be equivalent to digits. If a cell was either a black or a white (state), we could rename them "1" and "0" or "+" and "−". The overall behavior of each cell is a consequence of a rule where the next state of each cell depends on some function of the neighborhood cells; thus, the underlying mechanism is some kind of cellular automata. A cellular automaton is a kind of digital computer; it is discrete and deterministic. In a way, we could envision the whole universe as a system of cellular automata, a gigantic digital computer working based on a "general rule".

On the other hand, Fredkin assumes that if this general rule, the answer to everything going on in nature, were known, we could use it in two ways: analytically, as any mathematical equation, and through simulation, where we use computers or specially built cellular automaton machines. The usefulness of the simulation will depend on the scale. If the scale is the right size, we could calculate everything in our universe, its exact present state. To be practical, the scale of the rule (of the cellular automaton) must not be too far below the scale of the phenomena we wish to model.

1.6 *Discuss the benefits of computer simulation as an alternative method to science and engineering, complementing the traditional approaches, theoretical modelling and experiments. Give examples from your own area of interest and outline the benefits of computer simulation for your specific examples.*

Discussion points:

Computer simulation is as good as the model it is based on.
Computer simulation, no matter how complex and flexible, cannot replace experiments and tests.
Computer simulations enable the understanding of theoretical models, especially when the problems are too complex or difficult to solve using analytical methods.
A good computer simulation is simpler and cheaper to build, easier to modify; it is simple to understand and to convey its results, The key parameters of the theoretical model can be varied and the effects can be observed and understood from the computer simulation output.

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[4] Ed. Fredkin. *Digital Machines: An Informational Process Based on Reversible Universal Cellular Automata.* Physica D 45:254-270, 1990 (See on line at http://digitalphilosophy.org together with *A New Cosmology - On the Origin of the Universe*).

[5] J. Brown. *The Quest for Quantum Computer.* Simon and Schuster, New York, N.Y., 1999.

The optimal values of key parameters can be selected and used in the design of experiments or prototypes.

1.7 *Relate the number of states of a system subject to computer simulation to the execution time and the space complexity of the algorithms used for simulation.*

Solution:

Given a system with n states, an algorithm simulating this system has the following characteristics:

Time complexity: $\mathcal{O}(n)$

Space complexity: $\mathcal{O}(n^2)$

The time and space complexity of the simulation algorithm is *at least* $\mathcal{O}(n)$.

(See Section 1.3 for a detailed discussion.)

1.8 *Is it feasible to simulate a quantum system using a classical computer? Justify your answer.*

Discussion points:

In 1982, Richard Feynman had pointed out that there seemed to be essential difficulties in simulating quantum systems on classical computers. Deterministic simulations of quantum phenomena using classical computers requires a number of resources that scale exponentially with the number of degrees of freedom. The probabilistic simulation of certain quantum problems is limited by the so-called sign or phase problem, a problem believed to be of exponential complexity. A quantum computer is supposed to mimic physical processes exactly the same as Nature.

This is what Feynman had to say in his 1982 paper[6]: "Can physics be simulated by a universal computer? ... the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics ... the full description of quantum mechanics for a large system of R particles ... has too many variables, it **cannot be simulated** with a normal computer with a number of elements proportional to R ... but it can be simulated with quantum elements. ... Can quantum system be probabilistically simulated with a classical (probabilistic, I'd assume) universal computer? ... If you take the computer to be the classical kind I've described so far ... the answer is certainly, No!"

1.9 *Consider the multiple beam splitter experiment discussed in Section* 1.5. *Assuming that a beam splitter tosses a fair ternary coin to decide if a photon should be transmitted, reflected, or absorbed, what are the probabilities of the five detectors in Figure 1(b) detecting a photon? If the experiment is carried out* 100,000 *times, how many counts will each detector register?*

Solution:

At each beam splitter the probability to be transmitted, p_t, the probability to be reflected, p_r, and the probability to be absorbed, p_a are equal :

$$p_{\mathrm{t}} = \frac{1}{3}, \quad p_{\mathrm{r}} = \frac{1}{3}, \quad p_{\mathrm{a}} = \frac{1}{3}.$$

The number of counts recorded by each detector is $N_{\mathrm{count}} = N \times p_{\mathrm{detect}}$, where N is the number of photons sent through the system.

[6]R. P. Feynman. *Simulating Physics with Computers.* Int. J. Theoretical Physics 21:467–488, 1982.

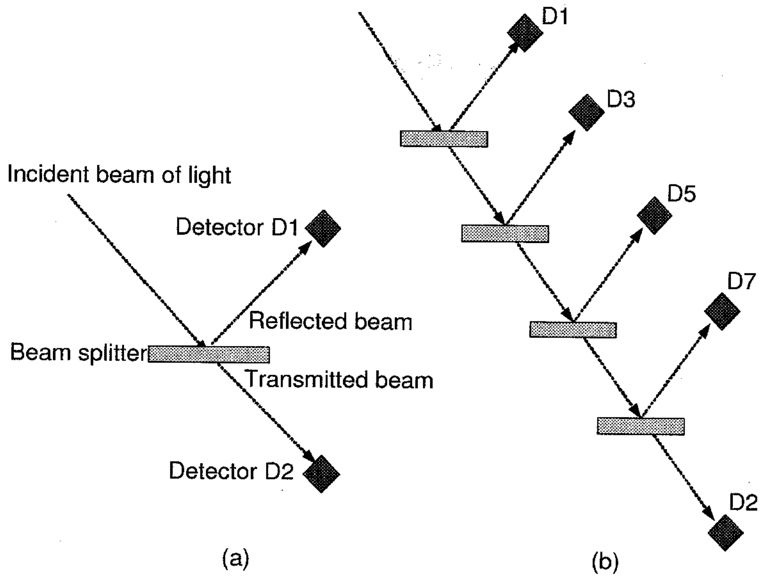

Figure 1: (a) One beam splitter. We send a single photon and observe that either detector D1 or detector D2 senses the photon. Repeating the experiment many times, we observe that each one of the two detectors records about the same number of events. (b) Cascaded beam splitters. All detectors have a chance of detecting the single photon sent in one experiment. The cascaded beam splitter experiment convinces us to dismiss the idea that a photon carries a "gene" and exhibits a deterministic behavior.

$$\begin{array}{ll}
D_1: & p_{\text{detect}} = p_{\text{r}} = \frac{1}{3}, \quad N_{\text{count}} \simeq 30,000 \\
D_3: & p_{\text{detect}} = p_{\text{t}} \times p_{\text{r}} = \frac{1}{(3)^2}, \quad N_{\text{count}} \simeq 10,000 \\
D_5: & p_{\text{detect}} = p_t \times p_t \times p_r = \frac{1}{(3)^3}, \quad N_{\text{count}} \simeq 3000 \\
D_7: & p_{\text{detect}} = p_t \times p_t \times p_t \times p_r = \frac{1}{(3)^4}, \quad N_{\text{count}} \simeq 1000 \\
D_2: & p_{\text{detect}} = p_t \times p_t \times p_t \times p_t = \frac{1}{(3)^4}, \quad N_{\text{count}} \simeq 1000
\end{array}$$

1.10 *Draw the four diagrams showing the path of a photon emitted by* S2 *in Figure* 1.8 *in Section* 1.9 *and calculate the probability of a photon from* S2 *to reach* D1 *and* D2.

Solution:

S2 $\rightarrow$ D1 : reflection at BS1 and transmission at BS2 or transmission at BS1 and reflection at BS2.

$$p_{\text{S2}\rightarrow\text{D1}} = p_{\text{RT}} + p_{\text{TR}} = (-\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = 0$$

S2 $\rightarrow$ D2 : reflection at BS1 and reflection at BS2 or transmission at BS1 and transmission at BS2.

$$p_{\text{S2}\rightarrow\text{D2}} = p_{\text{RR}} + p_{\text{TT}} = (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = 1$$

1.11 *Consider the experiment discussed in Section* 1.9 *and illustrated in Figure 2. Assume that a beam of electrons crosses the paths of the photons coming from sources* S1 *and* S2 *before they reach the beam splitter* BS1. *How will the outcome of this experiment be affected?*

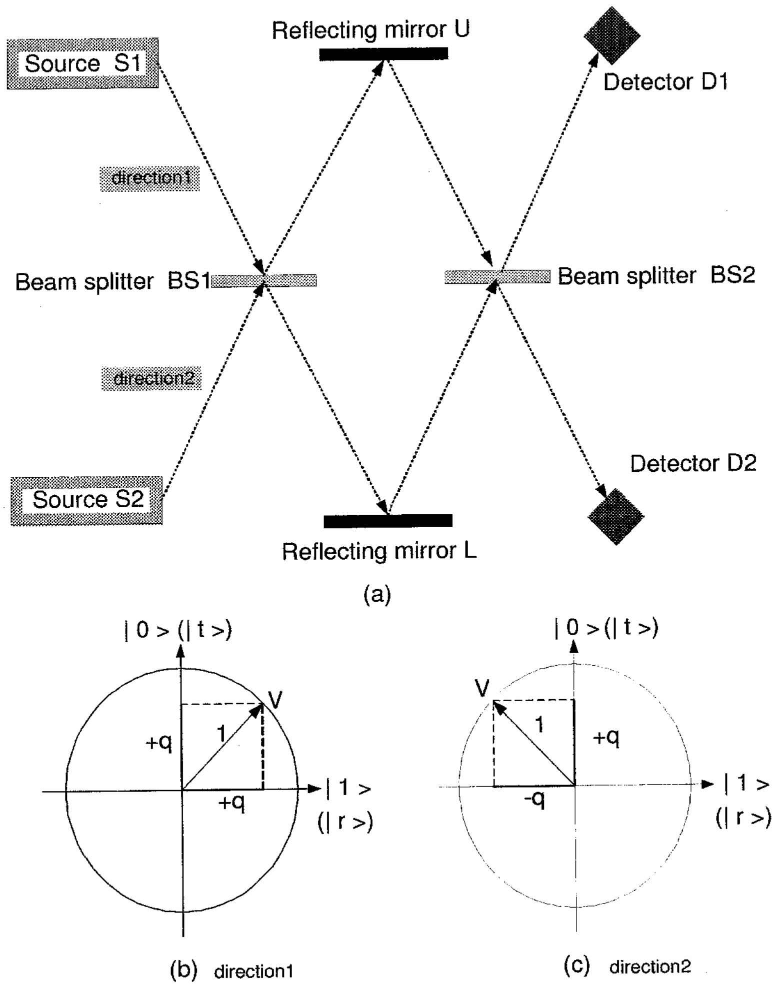

Figure 2: (a) Two sources, S1 and S2, generate photons, one at a time. There are two beam splitters, BS1 and BS2 and two detectors, D1 and D2. The beam splitters BS1 and BS2 are constructed so that the probability of a photon being reflected is equal to the probability of the photon being transmitted, $q = 1/\sqrt{2}$. We observe experimentally that all photons generated by S1 are detected by D1, none reaches D2. Conversely, all photons generated by S2 are detected by D2. The experimental observations are consistent with the superposition probability rule. (b) The state vector of a photon coming to one of the beam splitters from `direction1` is described by the vector $| \psi_1\rangle = \alpha_0 \mid 0\rangle + \alpha_1 \mid 1\rangle$. The projection of this state vector on $| 0\rangle$ is the probability amplitude of the event that the photon is transmitted, $\alpha_0 = +q$. The probability amplitude of a photon being reflected is $\alpha_1 = +q$. c) The state vector of a photon coming to one of the beam splitters from `direction2` is described by the vector $| \psi_2\rangle = \alpha_0 \mid 0\rangle - \alpha_1 \mid 1\rangle$. The projection of this state vector on $| 0\rangle$ is the probability amplitude of the photon being transmitted, $\alpha_0 = +q$. The projection on $| 1\rangle$ is the probability amplitude of a photon being reflected, $\alpha_1 = -q$.

Solution

The photons are scattered by the electrons in all directions. Some photons coming from S1 will not reach BS1 and appear as coming from S2 towards the mirror U, or will come to BS2 and then to D1 or D2 without interacting with BS1 or the mirror U. Photons coming from

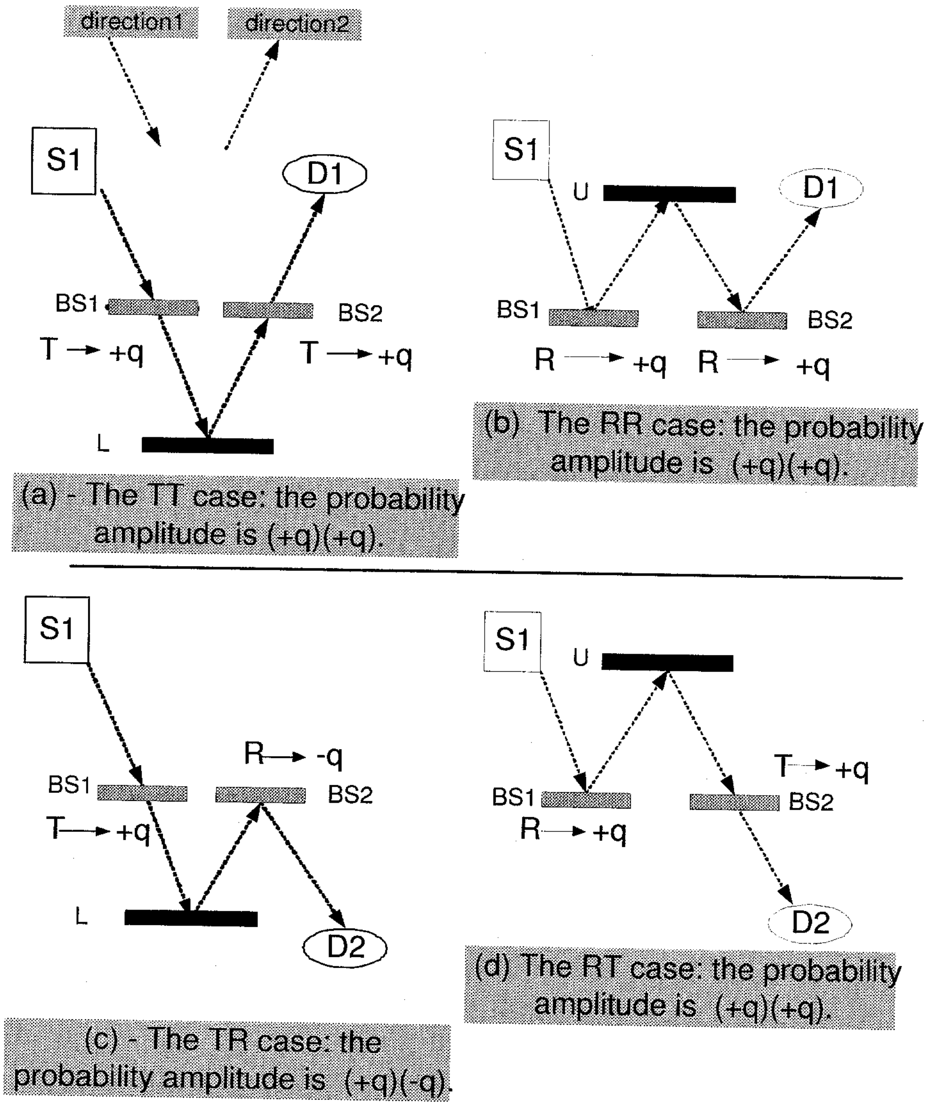

Figure 3: There are four possible events when a photon is emitted by `S1`. (a) The probability amplitude of a photon coming from `direction1` to be transmitted by `BS1` is $+q$ and the probability amplitude of a photon coming from `direction2` to be transmitted by `BS2` is also $+q$. Consequently, TT – the event that the photon is transmitted by both beam splitters has a probability amplitude $(+q)(+q) = q^2$. (b) The probability amplitude of a photon coming from `direction1` to be reflected (by `BS1`) is $+q$ and the probability amplitude of a photon coming also from `direction1` to be reflected by `BS2` is also $+q$. It follows that the probability amplitude of RR is again $(+q)(+q) = q^2$. (c) TR – the photon is transmitted by the first beam splitter and reflected by the second with a probability amplitude $-q^2$. (d) RT – the photon is reflected by the first beam splitter and transmitted by the second with a probability amplitude q^2. As before, $q = 1/\sqrt{2}$. The probability amplitude of a photon emitted by `S1` to reach `D1` is the sum of the two amplitudes of TT and RR, it is equal to $2q^2$. The probability of a photon emitted by `S1` to reach `D1` is $p_{S1 \to D1} = (2q^2)^2 = 4q^4 = 4(1/\sqrt{2})^4 = 1$.

`S2` and scattered by the electrons will not reach `BS1`, fly directly to the mirror `L` or to `BS2` and then to `D1` or `D2`. The probabilities of reaching either `D1` or `D2` will be modified.

1.14 *Consider a setup similar to that in Sections* 1.9 *and* 1.11. *Show that for the case of an experimental setup with an odd number of beam splitters,* $1, 3, 5, \ldots (2k+1) \ldots$, *the result is that presented in this section*

$$p_{S1\to D1} = p_{S1\to D2} = 1/2 \qquad p_{S2\to D1} = p_{S2\to D2} = 1/2.$$

For the case of an even number of beam splitters, $2, 4, 6, \ldots (2k) \ldots$ *show that*

$$p_{S1\to D1} = 1 \qquad p_{S1\to D2} = 0 \qquad p_{S2\to D1} = 0 \qquad p_{S2\to D2} = 1.$$

Solution:

1. Odd number of beam splitters: See Section 1.11 for a detailed discussion.
2. Even number of beam splitters: (**Hint:** follow the line of the discussion presented in Section1.11 for the odd number of beam splitters)

1.15 *Consider an experimental setup similar to that in Section* 1.9 *and assume that the state vectors of photons originating from* `direction1` *and* `direction2`, *respectively, are*

$$\mid \psi_1\rangle = +q \mid 0\rangle + q \mid 1\rangle \qquad \mid \psi_2\rangle = -q \mid 0\rangle + q \mid 1\rangle$$

Show that

$$p_{S1\to D1} = 0 \qquad p_{S1\to D2} = 1 \qquad p_{S2\to D1} = 1 \qquad p_{S2\to D2} = 0.$$

Solution:

`S1` $\to$ `D1` : transmission at `BS1` and transmission at `BS2` or reflection at `BS1` and reflection at `BS2`.

$$p_{\mathrm{S1\to D1}} = p_{\mathrm{TT}} + p_{\mathrm{RR}} = (\frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = 0$$

`S1` $\to$ `D2` : transmission at `BS1` and reflection at `BS2` or reflection at `BS1` and transmission at `BS2`.

$$p_{\mathrm{S1\to D2}} = p_{\mathrm{TR}} + p_{\mathrm{RT}} = (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = 1$$

`S2` $\to$ `D1` : reflection at `BS1` and transmission at `BS2` or transmission at `BS1` and reflection at `BS2`.

$$p_{\mathrm{S2\to D1}} = p_{\mathrm{RT}} + p_{\mathrm{TR}} = \frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}) + (-\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = -1$$

`S2` $\to$ `D2` : reflection at `BS1` and reflection at `BS2` or transmission at `BS1` and transmission at `BS2`.

$$p_{\mathrm{S2\to D2}} = p_{\mathrm{RR}} + p_{\mathrm{TT}} = (\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) + (-\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) = 0$$